

Philosophy through Geometry

Merlin CCC, March 2022

Readings for Week 2: Limit & The Unlimited [Or, the Bound & the Infinite]

Reading 1: Euclid, *Elements*, Book I.¹

DEFINITIONS

1. A **point** is that which has no part.
2. A **line** is breadthless length.
3. The extremities of a line are points.
4. A **straight line** is a line which lies evenly with the points on itself.
5. A **surface** is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A **plane surface** is a surface which lies evenly with the straight lines on itself.
8. A **plane angle** is the inclination to one another of two lines in a plane, which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called **rectilineal**.
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is **right**, and the straight line standing on the other is called a **perpendicular** to that on which it stands.
11. An **obtuse angle** is an angle greater than a right angle.
12. An **acute angle** is an angle less than a right angle.
13. A **boundary** is that which is an extremity of anything.
14. A **figure** is that which is contained by any boundary or boundaries.
15. A **circle** is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;
16. And the point is called the **centre** of the circle.
17. A **diameter** of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.
18. A **semicircle** is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.
19. **Rectilineal figures** are those which are contained by straight lines, **trilateral figures** being those contained by three, **quadrilateral** those contained by four, and **multilateral** those contained by more than four straight lines.

¹ Translated by Sir Thomas L. Heath.

20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle** that which has two of its sides alone equal, and a **scalene triangle** that which has its three sides unequal.
21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, an **obtuse-angled triangle** that which has an obtuse angle, and an **acute-angled triangle** that which has its three angles acute.
22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** that which is right-angled but not equilateral; a **rhombus** that which is equilateral but not right-angled; and a **rhomboid** that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called **trapezia**.
23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES

Let the following be postulated:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes² are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

² The “whole” is that which results from the addition of the two parts.

Reading 2A: Aristotle, *Metaphysics* A.5, 987a13-21.³

The Pythagoreans said that there are two principles. But this much they added, which is peculiar to them, that they did not think the finite, and the infinite, and the one, were certain other natures, such as fire, or earth, or any similar thing; but they were of the opinion, that the infinite itself, and the one itself, are the essence of these things of which they are predicated: and hence they asserted that number is the essence of all things. After this manner, therefore, they unfolded their opinion respecting these things, and began to speak about what a thing is, and to define...

Reading 2B: Aristotle, *Metaphysics* A.5, 986a22-26.⁴

Others of these [Pythagoreans] assert that there are ten principles, which are denominated according to co-ordination [i.e., in pairs], thus:

Bound [a.k.a. Limit]	Infinite [a.k.a. Unlimited]
Odd	Even
One [Unity]	Multitude [Many]
Right Hand	Left Hand
Masculine	Feminine
Quiescent [That Which is at Rest]	That Which is in Motion
Straight	Curved
Light	Darkness
Good	Evil
Square	Oblong

Reading 3: Plato, *Philebus* 23e–26c.⁵

SOCRATES But let us begin cautiously, and endeavour to lay down right principles.

PROTARCHUS What principles do you mean?

SOC. All things which are now in the universe let us divide into two sorts, or rather, if you please, into three.

PROT. You should tell us what difference between things it is, with respect to which you make that division.

SOC. Let us reassume some things which have been already mentioned.

PROT. What things?

SOC. The God, we said, has exhibited the infinite, and also the bound of beings.

³ Translated by Thomas Taylor; slightly modified.

⁴ Translated by Thomas Taylor; modified, with additions in brackets.

⁵ Translated by Thomas Taylor; slightly modified.

PROT. Very true.

SOC. Let us take these for two of the species of things; and for a third let us take that, which is composed of those two mixed together. But I deserve, methinks, to be laughed at for pretending thus to distinguish things, and to enumerate their several species.

PROT. Why so, my good friend?

SOC. A fourth kind appears to have been omitted by me.

PROT. Say, What?

SOC. Of that commixture, the combination of the former two, consider the cause: and beside those three species, set me down this cause for a fourth.

PROT. Will you not want a fifth species too, for a cause of disunion and separation?

SOC. Perhaps I may; but not, I believe, at present. However, should there be occasion for it, you will pardon me, if I go in pursuit of a fifth.

PROT. Certainly.

SOC. Of these four species, then, in the first place dividing the three, and perceiving that two of these, when both are divided, and their divisions separated, are, each of them, many;- then, gathering together the many of each, and uniting them again, let us endeavour to understand in what manner each of them is, at the same time, one and many.

PROT. If you would but express your meaning more plainly, I might, perhaps, apprehend it.

SOC. I mean, then, by the two, which I propose to be now considered, the same which I mentioned at the first; one of them the infinite, and the other bound. That the infinite is, in some manner, many, I will attempt to show: and let bound wait a while.

PROT. It shall.

SOC. Give me now your attention. It is, I confess, a difficult and doubtful thing, that, which I would have you to consider. Consider it, however. First, with regard to hotter and colder, in things, see if you can think of any bound. Or would not the more and the less, residing in the kinds themselves of things, hinder an end from being fixed to them, so long as they reside there? For, if ever they receive an end, their very beings are then also brought to an end.

PROT. Most certainly true.

SOC. And in speaking of either the colder or the hotter of any two things, we constantly attribute to them the more and the less.

PROT. And very much so.

SOC. Reason then constantly suggests to us that the colder and the hotter have no end: and being thus without any end, they are altogether boundless.

PROT. I am strongly inclined to agree with you, Socrates, in this point.

SOC. Well have you answered, my friend Protarchus; and well have you reminded me, that the strongly, which you mentioned, and the faintly, have the same power as the more and the less. For, wherever they reside, they suffer not any thing to be just so much; but infusing either the more intense or the more remiss into every action, they always produce in it either the more or the less; while the "just so much" flies away and vanishes from before them. For, as it was just now observed, were they not to drive away the just so much, did they permit this, and the moderate, to enter into the regions

of the more and the less, or of the intense and the remiss, these very beings must quit their own places: because, if they admitted the just so much, the hotter and the colder would be gone. For the hotter, and in like manner the colder, is always advancing forward, and never abides in the same spot: but the just so much stops, and stays, having finished its progress. Now, according to this reasoning, the hotter must be boundless; and so must also be the colder.

PROT. So it appears indeed, Socrates. But, as you rightly said, it is not easy to apprehend these things. Questions, however, relating to them, again and again repeated, might perhaps show that the questioner and the respondent were tolerably well agreed in their minds concerning them.

SOC. You say well: and we should try so to do. But at present, to avoid lengthening out this argument, by enumerating every infinite, consider, whether we may take this for the characteristic mark of the nature of all infinities.

PROT. What mark do you mean?

SOC. Whatever things appear to us to be increasing or diminishing, or to admit of intenseness and remission, or the too much, and all other such attributes, we ought to refer all these to the genus of the infinite; collecting, as it were, all of them in one, agreeably to what was before said; that whatever things were divided and separated we ought to assemble together and combine, as well as we are able, affixing to all of them the mark of some one nature;- if you remember.

PROT. I remember it well.

SOC. Every thing, then, which rejects all such attributes, and admits only such as are quite the contrary, - in the first place, the equal and equality, and, after the equal, the double, and every other relation which one number bears to another, and one measure to another, - all these things, I say, in summing up, and referring them to bound, think you not that we should do right? or how say you?

PROT. Perfectly right, O Socrates.

SOC. Well: but the third thing made up, and consisting of the other two, what characteristic shall we assign to this?

PROT. You, as I presume, will show it to me.

SOC. Divinity indeed may; if any of the Gods will hearken to my prayers.

PROT. Pray, then, and survey.

SOC. I survey: and some God, O Protarchus, is now, methinks, become favourable to us.

PROT. How do you mean? and by what sign do you know it?

SOC. I will tell you in plain words: but do you follow them closely.

PROT. Only speak.

SOC. We mentioned just now the hotter and the colder; did we not?

PROT. We did.

SOC. To these then add the drier and the moister; the more numerous and the fewer; the swifter and the slower; the larger and the smaller; and whatever things beside, in our late account of them, we ranked under one head, - that which admits of the nature of the more and the less.

PROT. You mean the infinite.

SOC. I do: and mingle together with this that which we spoke of next afterward, - the race of bound.

PROT. What race do you mean?

SOC. Those things which we did not (as we ought to have done) assemble together under one head, in the same manner as we assembled together the race of the infinite. But you will now, perhaps, do what was then omitted. And when both the sorts are assembled, and viewed together, the race of bound will then become manifest.

PROT. What things do you speak of? and how are they to be assembled?

SOC. I speak of that nature in which are comprised the equal and the double; and whatever else puts an end to contest between contrary things; and, introducing number, makes them to be commensurate one with another, and to harmonize together.

PROT. I apprehend your meaning to be, that, from the commixture of those two, a certain progeny will arise between them in every one of their tribes.

SOC. You apprehend me rightly.

PROT. Relate then the progeny of these commixtures.

SOC. In diseases, does not the right commixture of those two produce the recovery of health?

PROT. Entirely so.

SOC. And in the acute and the grave, in the swift also and the slow, which are all of them infinite, does not the other sort, received among them, and begetting bounds, constitute the perfection of all the Muse's art?

PROT. Certainly so.

SOC. And in weather excessively either cold or hot, does not the entrance of that other kind take off the excess, the vehement, and the infinite, - generating in their stead, not only the moderate and the measured, but symmetry also, and correspondence between their measures?

PROT. Without dispute.

SOC. And do not propitious seasons, and all their fair productions, arise to us from hence, from the mixture of things which are infinite with things which have a bound?

PROT. Doubtless.

SOC. A thousand other things I forbear to mention; as, for instance, strength and beauty, the attendants upon health of body; and in the soul other excellencies, very many and very noble. For Aphrodite herself, O good Philebus! observing lawless lust, and all manner of vice every where reigning, the love of pleasure being in all men boundless, and their desires of it insatiable, she herself established a law and an order, setting bounds to pleasure and desire. This you said was to lessen and to impair pleasure; but I maintain that, on the contrary, it preserved pleasure from decay. And you, Protarchus! what think you of it?

PROT. For my part, I am entirely of your mind, Socrates.